QUANTUM QUENCH & AdS/CFT

WITH

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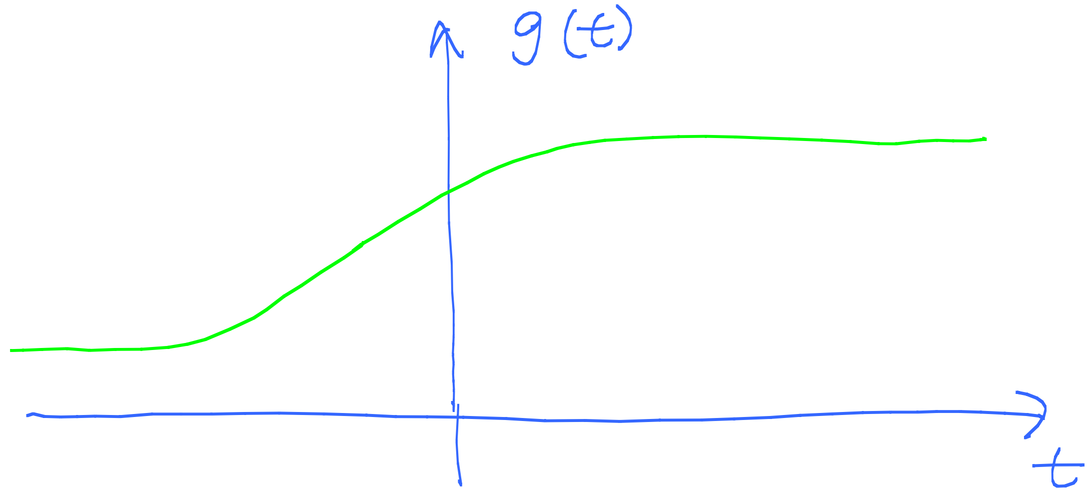
THE PROBLEM

SUPPOSE A QUANTUM SYSTEM HAS A TIME DEPENDENT COUPLING CONSTANT



IF WE START IN THE GROUND STATE AT $t \to -\infty$

WHAT IS THE NATURE OF THE STATE SUBSEQUENTLY ?

DOES IT RESEMBLE A THERMAL STATE AT LATE TIMES ?

- FOR FIELD THEORIES THIS PROBLEM HAS RECENTLY BECOME INTERESTING — COLD ATOM EXPERIMENTS CAN NOW ACHIEVE SUCH CONDITIONS EXPERIMENTALLY

- THEORETICALLY – THIS HAS BEEN STUDIED MOSTLY IN TWO EXTREME SITUATIONS

  - SLOW EVOLUTION
  - RAPID QUENCH

BOTH THESE SITUATIONS ARE PARTICULARLY INTERESTING IF THE COUPLING PASSES THROUGH A CRITICAL POINT WHERE THE GAP VANISHES

IF SYSTEM HAS A GAP $\Delta(g(t))$
AND

$$\dot{g}(t) \ll \frac{1}{\Delta(g(t))}$$

FOR ALL TIMES t, ADIABATICITY WILL HOLD

— SYSTEM WILL REMAIN IN THE INSTANTANEOUS GROUND STATE — UP TO EXPONENTIALLY SMALL CORRECTIONS

HOWEVER, NEAR A CRITICAL POINT THE ENERGY GAP VANISHES

$$\boxed{\Delta[g(t)] \sim (g-g_c)^{z\nu}}$$

$$\boxed{\xi[g(t)] \sim (g-g_c)^{-\nu}}$$

AS $t \to 0$ ADIABATICITY WILL FAIL — SYSTEM WILL GET EXCITED

FOR A LARGE CLASS OF SYSTEMS THE PROBABILITY FOR EXCITATION MAY BE ESTIMATED BY SIMPLE SCALING ARGUMENTS

ADIABATICITY WILL FAIL WHEN

$$\boxed{\frac{d\Delta}{dt} \sim \Delta^2}$$

FOR A TIME DEPENDENCE OF THE FORM

$$\boxed{g - g_c = v t}$$

ADIABATICITY FAILS AT $t = t_*$

$$\Delta \sim \frac{1}{t_*} \Rightarrow \boxed{\Delta(t_*) \sim |v|^{\frac{z\nu}{z\nu+1}}}$$

AT THIS TIME CORRELATION LENGTH IS

$$\boxed{\xi(t_*) \sim |v|^{-\frac{\nu}{z\nu+1}}}$$

z = DYNAMICAL CRITICAL EXPONENT

ASSUMING SCALING HOLDS — THE EXPECTATION VALUE OF SOME OPERATOR WITH (LENGTH) DIMENSION $-x$ SHOULD SCALE AS

$$\boxed{\langle O \rangle \sim \xi^{-x}}$$

FOR EXAMPLE DENSITY OF QUASIPARTICLE EXCITATIONS SHOULD SCALE AS

$$\boxed{n \sim \xi^{-d} \sim |v|^{\frac{d\nu}{z\nu+1}}}$$

THIS IS KIBBLE-ZUREK SCALING

REASONABLE APPROXIMATION — FOR $-t_* < t < t_*$ SYSTEM MAY BE TREATED IN AN SUDDEN APPROXIMATION

RELAXATION

ONCE SYSTEM GETS EXCITED — IT SHOULD RELAX

- DOES IT RELAX TO A STATIONARY STATE?
- IF IT DOES — CAN THE STATIONARY STATE BE APPROXIMATED BY A THERMAL STATE?

A PURE STATE WILL OF COURSE NEVER BECOME THERMAL — QUANTUM RECURRENCE

HOWEVER THE STATE MAY RESEMBLE A THERMAL STATE FOR SOME SUBSET OF LOCAL OBSERVABLES

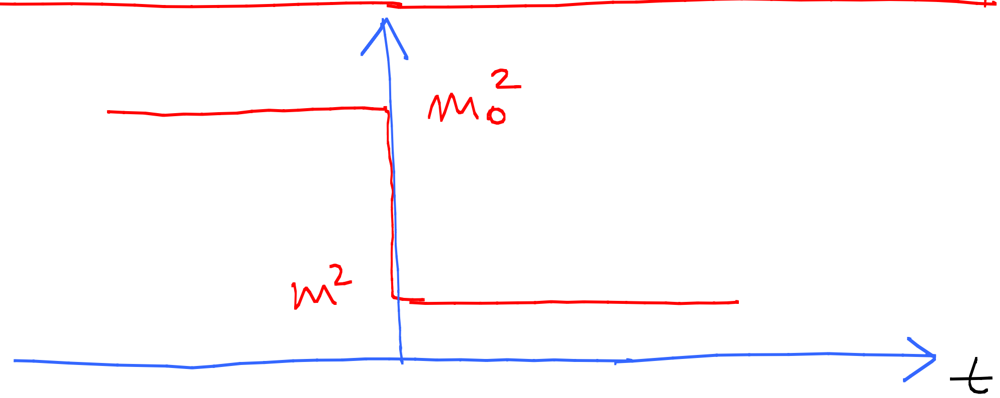
HOW DOES THE SYSTEM APPROACH THIS STATE?

RAPID QUENCH – FREE FIELDS

THIS PROBLEM CAN BE ADDRESSED IN THE RAPID QUENCH LIMIT

CONSIDER e.g FREE FIELD THEORY

$$\boxed{S = \frac{1}{2}\int d^dx\, dt \left[(\partial_t \phi)^2 - (\nabla\phi)^2 - m^2(t)\,\phi^2 \right]}$$



- PREPARE SYSTEM IN GROUND STATE FOR $t < 0$

- CALCULATE FEYNMAN PROPAGATOR FOR $t > 0$

$$\boxed{G(x, t_1; 0, t_2) = \int \frac{d^dk}{(2\pi)^d}\, G(k; t_1, t_2)\, e^{-i\vec{k}\cdot\vec{x}}}$$

$$\boxed{G(k;t_1,t_2) = \frac{1}{2\omega_k} e^{-i\omega_k|t_1-t_2|} + \frac{(\omega_k-\omega_{0k})^2}{4\omega_{0k}\omega_k^2}\cos\omega_k(t_1-t_2) + \frac{(\omega_k^2-\omega_{0k}^2)}{4\omega_{0k}\omega_k^2}\cos\omega_k(t_1+t_2)}$$

$$\omega_{0k}^2 = \vec{k}^2 + m_0^2$$
$$\omega_k^2 = \vec{k}^2 + m^2$$

SINCE THIS IS A FREE FIELD THEORY THE 2 POINT FUNCTION ENCODES ALL INFORMATION

Ref: Calabrese & Cardy
Sotiriadis & Cardy
Calabrese, Cardy & Sotoriadis

EQUAL TIME PROPAGATOR

(a) DEEP QUENCH

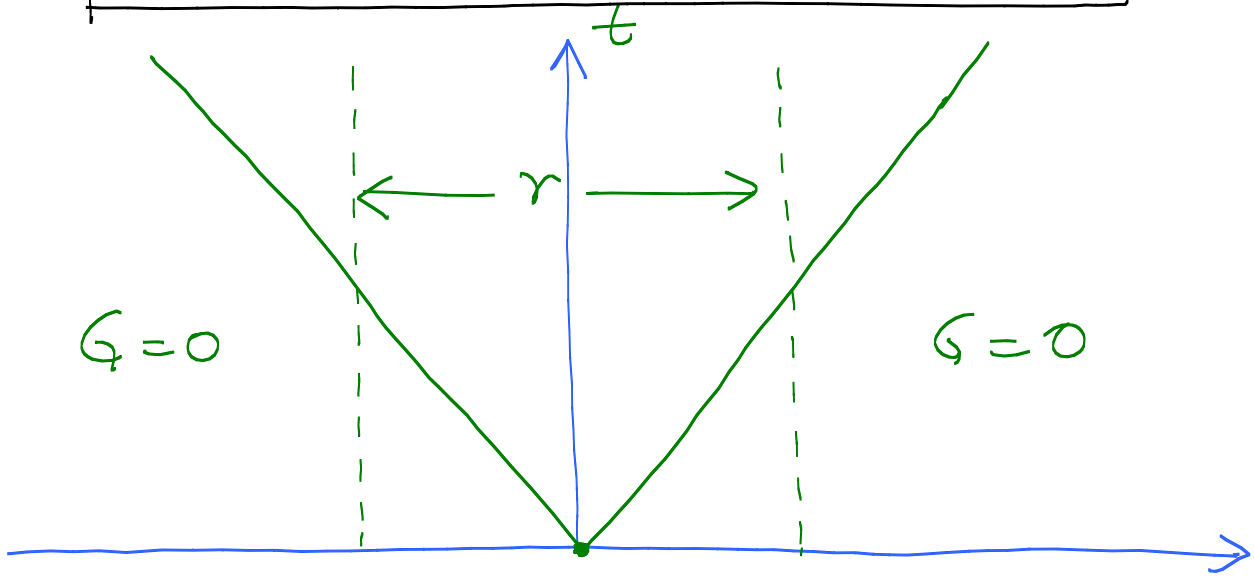
THIS IS THE LIMIT WHERE

$$\frac{m_0}{m} \gg 1$$

FOR $t_1 = t_2$ $\quad t, r \gg m_0^{-1}$

$$\boxed{G(k; t, t) = \frac{m_0}{4\omega_k^2}\left(1 - \cos 2\omega_k t\right)}$$

- $\boxed{G(r, t) = 0 \quad \text{FOR} \quad r > 2t}$



→ HORIZON EFFECT

QUENCH TO CRITICAL H

- FOR $m = 0$ — QUENCH TO A CRITICAL HAMILTONIAN

$$G(r,t) = \begin{cases} 0 & r > 2t \\ \frac{m_0}{8}(2t-r) & r < 2t \end{cases} \qquad d=1$$

$$G(r,t) = \begin{cases} 0 & r > 2t \\ \frac{m_0}{8\pi}\log\left[2t + \frac{1}{r}\sqrt{4t^2-r^2}\right] & r < 2t \end{cases} \qquad d=2$$

$$G(r,t) = \begin{cases} 0 & r > 2t \\ \frac{m_0}{16\pi r} & r < 2t \end{cases} \qquad d=3$$

FOR $d=3$ PROPAGATOR BECOMES STATIONARY EVEN FOR $m=0$

THE SAME IS TRUE FOR $d=1$

$$\boxed{\left\langle e^{iq\phi(0,t)} e^{-iq\phi(r,t)} \right\rangle = \begin{cases} e^{-\frac{q^2 m_0 t}{4}} & r > 2t \\ e^{-\frac{q^2 m_0 r}{8}} & r < 2t \end{cases}}$$

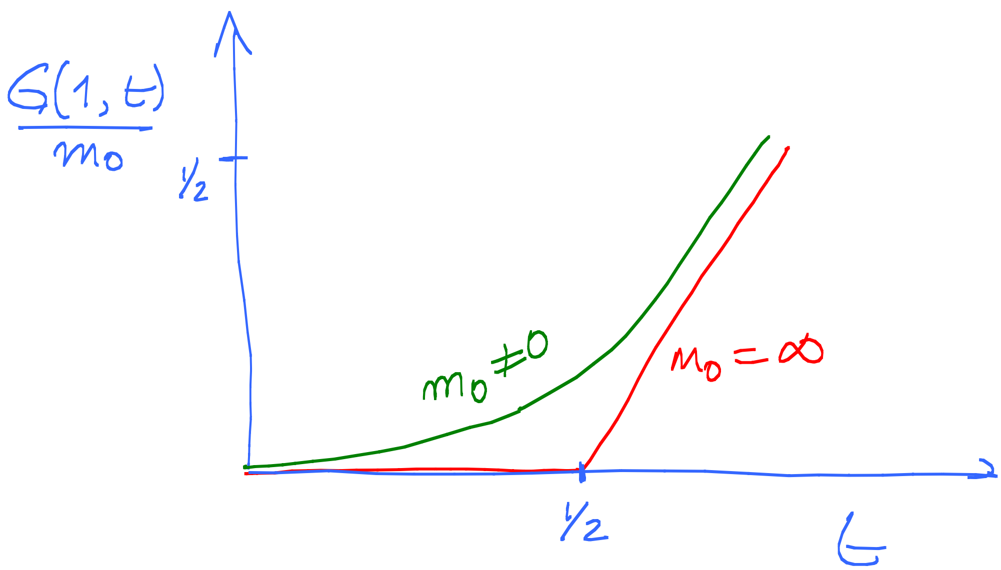
THUS CORRELATORS FOR CONFORMAL FIELDS

- DECAY EXPONENTIALLY IN TIME OUTSIDE HORIZON
- STATIONARY INSIDE HORIZON

THUS IN $m_0 \to \infty$ LIMIT ONE HAS INSTANTANEOUS "THERMALIZATION" FOR $d=1$ and $d=3$

(b) FINITE m_0

FOR FINITE m_0 THE HORIZON EFFECT IS SMOOTHENED OUT AS EXPECTED



RAPID QUENCH : 1+1 CFT

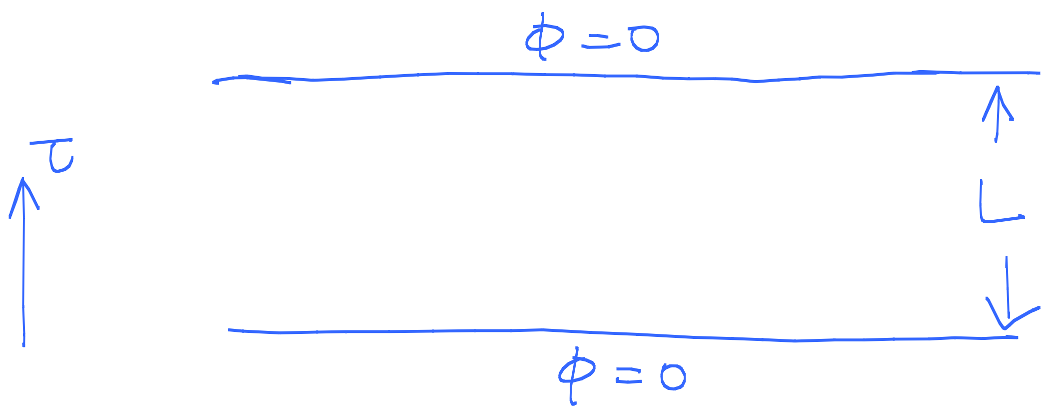
CONSIDER THE GENERAL EXPRESSION

$$\boxed{\begin{aligned} G(k; t_1, t_2) &= \frac{1}{2\omega_k} e^{-i\omega_k|t_1-t_2|} \\ &+ \frac{(\omega_k-\omega_{0k})^2}{4\omega_{0k}\omega_k^2} \cos\omega_k(t_1-t_2) \\ &+ \frac{(\omega_k^2-\omega_{0k}^2)}{4\omega_{0k}\omega_k^2} \cos\omega_k(t_1+t_2) \end{aligned}}$$

$$\omega_{0k}^2 = \vec{k}^2 + m_0^2$$

$$\omega_k^2 = \vec{k}^2 + m^2$$

THE POSITION SPACE PROPAGATOR IN EUCLIDEAN TIME $\tau = it$ IS IN FACT THE PROPAGATOR IN A SLAB GEOMETRY



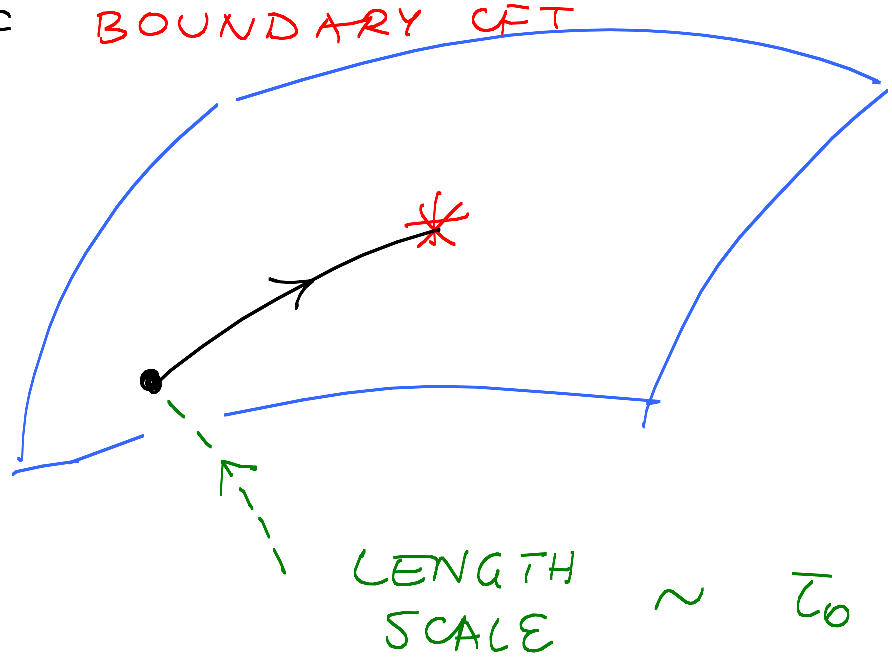
$$\boxed{\tanh\left(\frac{\omega_k L}{2}\right) = \begin{cases} \dfrac{\omega_k}{\omega_{0k}} & \omega_k < \omega_{0k} \\[2ex] \dfrac{\omega_{0k}}{\omega_k} & \omega_k > \omega_{0k} \end{cases}}$$

FOR $m_0 \gg m$

$$L \sim \frac{2}{m_0}$$

Calabrese & Cardy SHOWED THAT IN FACT, THIS REPRESENTATION FOR THE QUENCH PROPAGATOR IS VALID FOR ANY QFT WITH ARBITRARY LOCAL INTERACTIONS — IN ANY NUMBER OF DIMENSIONS

FOR QUANTUM QUENCH TO A 1+1 CFT FROM A POINT AWAY FROM THE FIXED POINT ONE CAN USE POWERFUL METHODS OF BOUNDARY CFT



$$\boxed{\langle \Phi(t) \rangle \sim \frac{1}{\tau_0^{x}}\, e^{-\frac{\pi x t}{2\tau_0}}}$$

$$\boxed{\langle \Phi(r,t)\, \Phi(0,t) \rangle \sim \begin{cases} e^{-\frac{\pi x t}{\tau_0}} & t < \frac{r}{2} \\ e^{-\frac{\pi x r}{2\tau_0}} & t > \frac{r}{2} \end{cases}}$$

x = CONFORMAL DIMENSION

RATIO OF RELAXATION TIMES FOR DIFFERENT OPERATORS ARE UNIVERSAL

$$\boxed{\frac{\tau^{(1)}_{relax}}{\tau^{(2)}_{relax}} = \frac{x_2}{x_1}}$$

"THERMALIZATION"

CONSIDER THE GENERAL EXPRESSION

$$\boxed{\begin{aligned} G(k;t_1,t_2) = {} & \frac{1}{2\omega_k} e^{-i\omega_k|t_1-t_2|} \\ & + \frac{(\omega_k-\omega_{0k})^2}{4\omega_{0k}\omega_k^2} \cos\omega_k(t_1-t_2) \\ & + \frac{(\omega_k^2-\omega_{0k}^2)}{4\omega_{0k}\omega_k^2} \cos\omega_k(t_1+t_2) \end{aligned}}$$

IF WE IGNORE THE LAST TERM

$$\boxed{G(k;t_1,t_2) = \frac{1}{2\omega_k} e^{-i\omega_k|t_1-t_2|} + \frac{(\omega_k-\omega_{0k})^2}{4\omega_{0k}\omega_k^2} \cos \omega_k (t_1-t_2)}$$

IS THE EXPRESSION FOR THERMAL PROPAGATOR WITH A MOMENTUM DEPENDENT TEMPERATURE

$$\boxed{G_\beta(k,t_1,t_2) = \frac{1}{2\omega_k} e^{-i\omega_k|t_1-t_2|} + \frac{2\cos \omega_k (t_1-t_2)}{\left(e^{\beta(k)\omega_k} - 1\right)\omega_k}}$$

$$\boxed{\beta(k) = \frac{2}{\omega_k} \log \frac{\omega_k+\omega_{0k}}{|\omega_k-\omega_{0k}|}}$$

- IF WE HAD A FINITE NUMBER OF ~~HARMONIC~~ OSCILLATORS, THE LAST TERM CANNOT BE IGNORED
  — IN FACT THIS IS CRUCIAL IN ENSURING THAT THE WAVEFUNCTION IS PERIODIC

- HOWEVER IN THE CONTINUUM & THERMODYNAMIC LIMIT
  — POSITION SPACE CORRELATOR

$$G(r, t_1, t_2) = \int \frac{d^d k}{(2\pi)^d} e^{i\vec{k}\cdot\vec{r}} G(k; t_1, t_2)$$

THE TIME-TRANSLATION NONINVARIANT TERM

$$\int \frac{d^d k}{(2\pi)^d} e^{i\vec{k}\cdot\vec{r}} \frac{\omega_k^2 - \omega_{0k}^2}{4\omega_{0k}\omega_k^2} \cos \omega_k (t_1 + t_2)$$

CANCELS IN THE $t_1, t_2 \to \infty$ LIMIT — DUE TO RAPID OSCILLATIONS

- THUS FOR LOCAL OBSERVABLES THE SYSTEM DISPLAYS SOME KIND OF THERMALITY

- OF COURSE A FREE THEORY CAN NEVER REALLY THERMALIZE — THIS IS REFLECTED BY THE FACT THAT THE TEMPERATURE IS MOMENTUM-DEPENDENT

- NEVERTHELESS, THE LONG DISTANCE PROPERTIES OF THE CORRELATOR IS GOVERNED BY $\beta(k=0)$

$$\boxed{\beta(k=0) = \frac{2}{m} \log \frac{m_0+m}{(m_0-m)}}$$

- FOR $m_0 \gg m$ (DEEP QUENCH)

$$\boxed{\beta(k=0) \simeq \frac{4}{m_0}}$$

- THE SAME EFFECTIVE TEMP IS OBTAINED USING USUAL BOGOLIUBOV TRANSFORMATION TECHNIQUES

- ONE WOULD EXPECT THAT IN A FULLY INTERACTING FIELD THEORY, VARIOUS MODES WILL REALLY THERMALIZE!

QUENCH IN INTERACTING QFT

- THERE IS NO GENERAL UNDERSTANDING OF THESE ISSUES IN STRONGLY INTERACTING CFTS — CASE BY CASE STUDY FOR SOME MODELS

- VERY FEW THEORETICAL TOOLS AVAILABLE — CRIES OUT FOR NEW TECHNIQUES

Reviews:

(i) J. Dziarmaga, 0912.4034

(ii) Polkovnikov, Sengupta, Silva & Vengalattore — Rev. Mod. Phys.

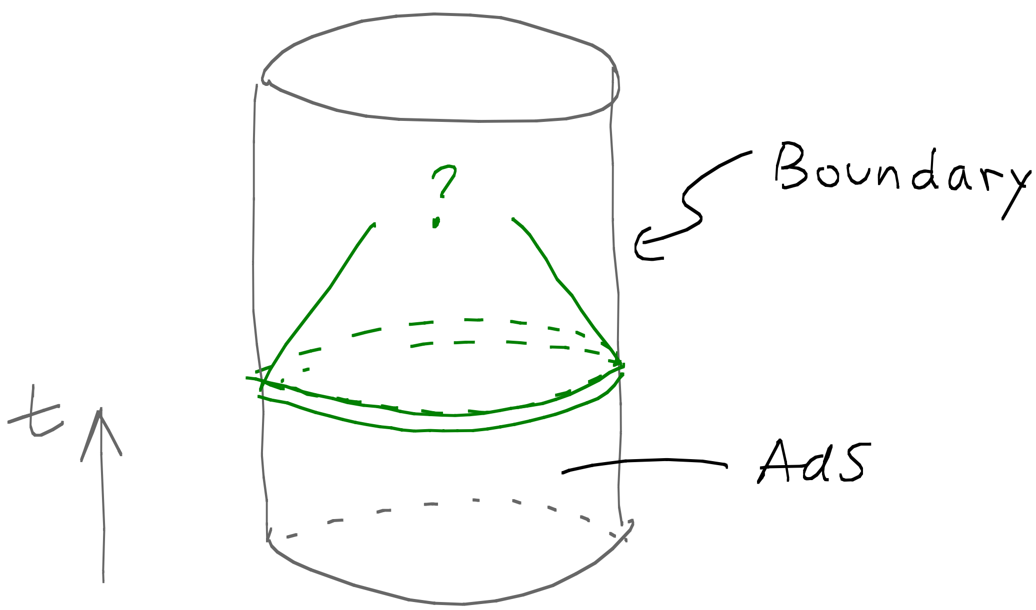
(iii) Sen, Mondal & Sengupta

$$\boxed{\text{CAN WE APPLY AdS/CFT TECHNIQUES TO STUDY THIS QUESTION ?}}$$

QUANTUM QUENCH & AdS/CFT

- THIS CLASS OF PROBLEM HAS BEEN ALREADY EXPLORED IN AdS/CFT
  – IN FACT, AdS/CFT WELL SUITED

TIME DEPENDENT COUPLING IN A QFT $\rightleftharpoons$ BOUNDARY CONDITION ON BULK FIELDS

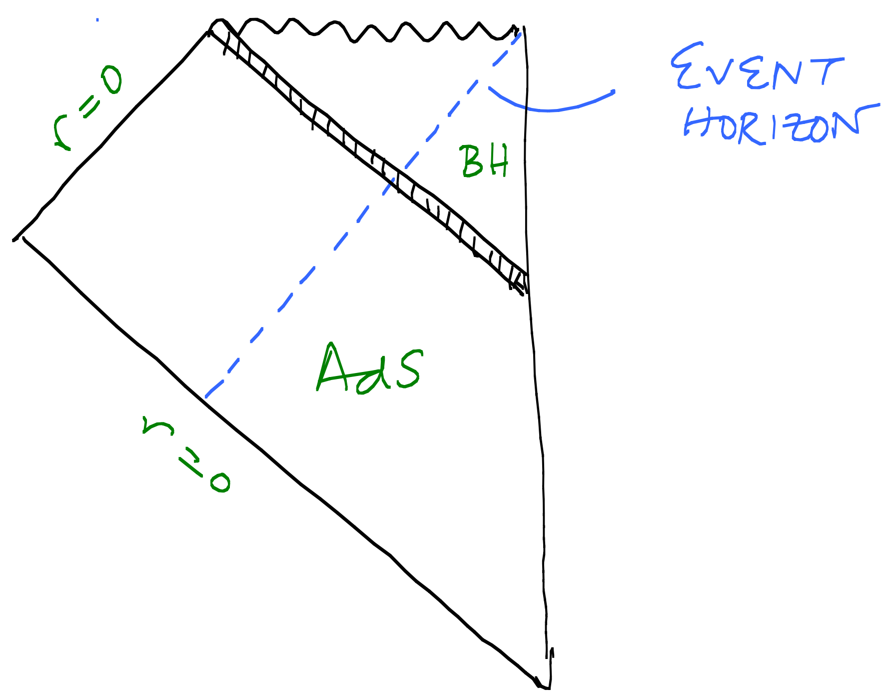


Question : HOW DOES A SOURCE DISTURBANCE AFFECT THE BULK ?

- IF THE COUPLING OF THE BOUNDARY THEORY ALWAYS REMAINS LARGE WE CAN ANSWER THIS QUESTION IN SUPERGRAVITY

Chesler & Yaffe : POINCARE PATCH BOUNDARY METRIC TIME DEP.

→ COLLAPSE TO A BLACK HOLE DUE TO THE RESULTING GRAVITATIONAL WAVE



$$\boxed{\begin{array}{l}\text{BLACK HOLE FORMATION}\\ \Rightarrow\ \text{THERMALIZATION OF THE}\\ \text{BOUNDARY THEORY}\end{array}}$$

Bhattacharya & Minwalla

- ENERGY INJECTION FROM BOUNDARY DUE TO
  - TIME DEP DILATON ($g_{YM}(t)$)
  - TIME DEP. METRIC

BOTH IN POINCARE & GLOBAL PATCH OF AdS

POINCARE $\Rightarrow$ BLACK HOLE FORMS FOR ANY RATE OF VARIATION

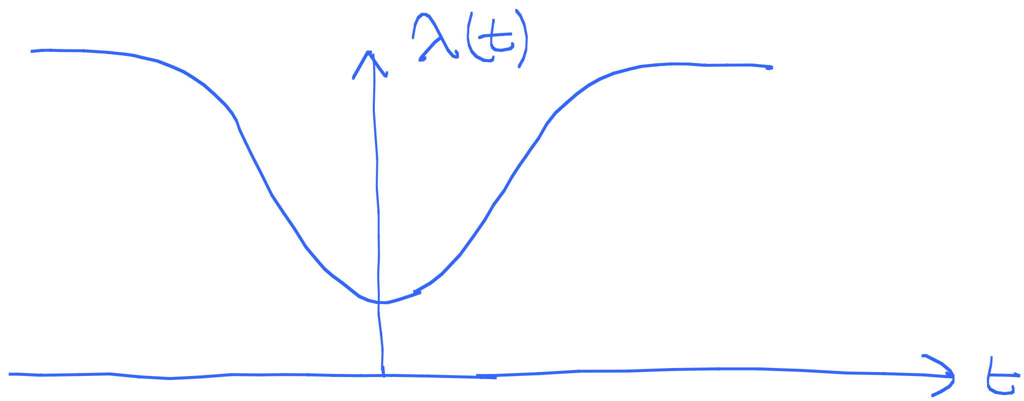
GLOBAL $\Rightarrow$ FOR SLOW VARIATION NO HORIZON

FOR FAST VARIATION FORMS

- IN THESE EXAMPLES, THE THERMALIZATION OCCURS VERY FAST — MUCH FASTER THAN THE SCALE OF EVENTUAL TEMPERATURE

— MAY HAVE SOMETHING TO DO WITH RAPID THERMALIZATION OBSERVED IN QUARK GLUON FLUID AT RHIC

WHEN THE BOUNDARY COUPLING BECOMES SMALL DURING TIME EVOLUTION



SUPERGRAVITY CANNOT BE TRUSTED ANY MORE IN THIS REGION

→ LARGE STRING FRAME CURVATURE

← PHYSICALLY SIMILAR TO A SPACELIKE SINGULARITY

IN GLOBAL AdS — FOR A VARIATION OF $\lambda = g_{YM}^2 N\ (t)$ SLOW COMPARED TO R_{AdS} — THE GAUGE THEORY CAN BE USED TO SAY A FEW THINGS ABOUT TIME EVOLUTION ACROSS THIS SINGULARITY

Awad, S.R.D, Narayan, Trivedi
Awad, S.R.D, Ghosh, Oh & Trivedi

S.R.D, T. Nishioka, T. Takayanagi

QUANTUM QUENCH & PROBE BRANES

- IN THE REST OF THE TALK WE WILL USE A DIFFERENT SETUP WHICH WILL TURN OUT TO BE USEFUL TO STUDY QUANTUM QUENCH ACROSS CRITICAL POINTS

- THIS INVOLVES INTRODUCTION OF A SMALL NUMBER N_f OF D-BRANES IN AdS SPACETIMES

- THE EFFECT OF THIS IS TO INTRODUCE HYPERMULTIPLETS IN DUAL FIELD THEORY

- THE HYPERMULTIPLET SECTOR GENERALLY LIVES ON A DEFECT IN THE BOUNDARY

WE WILL ALWAYS WORK IN THE STRONG COUPLING REGIME OF THE QFT $\rightarrow$ SUPERGRAVITY REGIME OF BULK

D5 – D3 SYSTEM

- CONSIDER e.g. D5 BRANES IN $AdS_5 \times S^5$

$AdS_5 \times S^5$ METRIC

$$\boxed{ds^2 = (r^2+y^2)(-dt^2+dx_1^2+dx_2^2+dx_3^2) + \frac{1}{r^2+y^2}\left(dr^2+r^2 d\Omega_2^2+dy^2+y^2 d\bar{\Omega}_2^2\right)}$$

D5 along $(t, r, x_1, x_2, \Omega_2)$

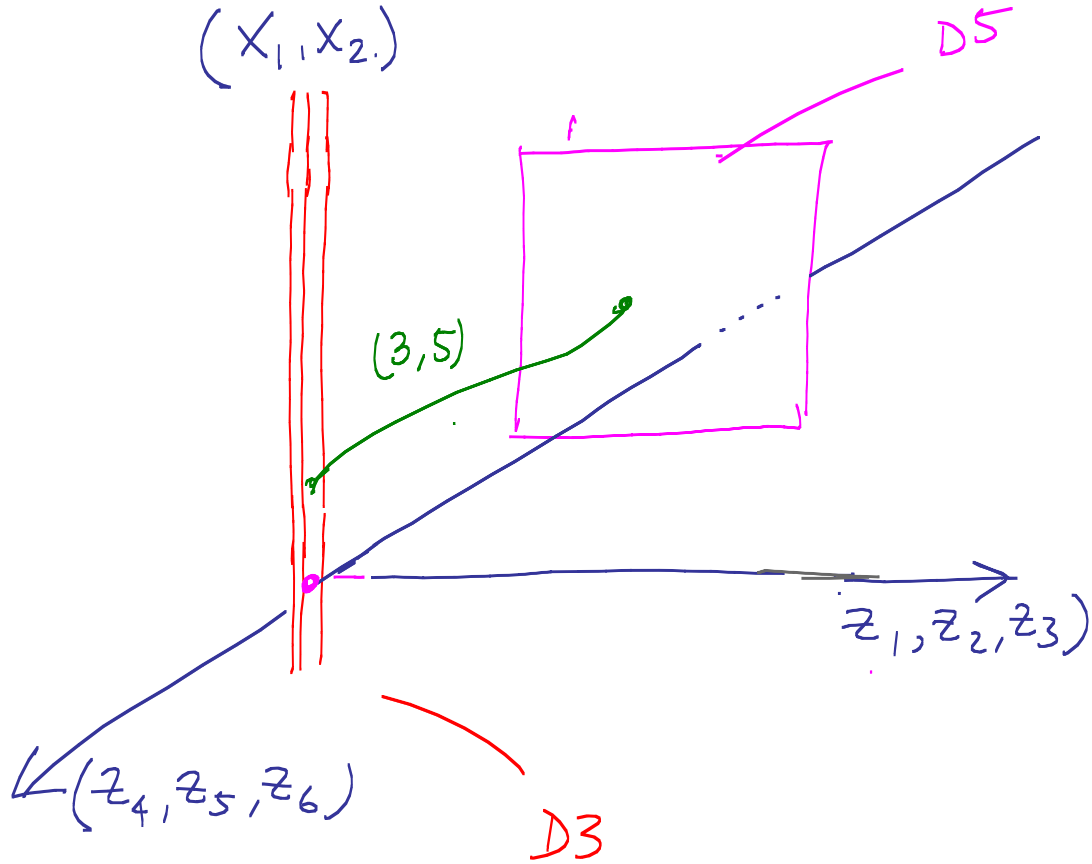
IN TERMS OF CARTESIAN COORDINATES TRANSVERSE TO THE D3 BRANES WHICH PRODUCE AdS_5

$$(z_1 \ldots\ldots z_6) \equiv (r, \Omega_2; y, \bar{\Omega}_2)$$

D3 : t, x_1, x_2, x_3

D5 : $t \quad x_1 \quad x_2 \qquad z_1, z_2, z_3$

TRANSVERSE COORDINATES $= x_3, \underbrace{z_4, z_5, z_6}_{y, \bar{\Omega}_2}$



N=4 SYM LIVES ON (t, x_1, x_2, x_3)
HYPERMULTIPLET FIELDS LIVE ON
$(t, x_1, x_2) \longrightarrow$ 2+1 DIM THEORY

ADJOINT FIELDS DUAL TO THE
SUPERGRAVITY FIELDS IN BULK

FUNDAMENTAL REP. FIELDS DUAL
TO D5 BRANE DEGREES
OF FREEDOM — DBI ACTION

- BULK SUGRA ACTION CONTROLLED BY

$$\frac{1}{G_5} = \frac{R^5}{g_s^2 \, l_s^8} = \frac{N_c^2}{R^3}$$

- DBI ACTION CONTROLLED BY

$$\frac{N_f}{g_s \, l_s^{p+1}} = \frac{N_f \, N_c}{R^4} \, l_s^{(p-3)}$$

THUS WHEN $N_f \ll N_c$ THE BRANE CAN BE CONSIDERED AS A PROBE

NOW THE ENTIRE DYNAMICS IS GIVEN BY THE DBI ACTION

— BACKREACTION ON BULK GEOMETRY CAN BE IGNORED

IN THE FIELD THEORY QUARK LOOPS CAN BE IGNORED

CONSIDER THE D5 PROBE BRANE

THE D5 BRANE CAN REMAIN AT A CONSTANT VALUE OF X_3 AND AT A CONSTANT VALUE OF THE ANGLES ON Ω_2

$\Rightarrow$ CONSIDER DBI ACTION FOR THE REMAINING TRANSVERSE COORDINATE $y(r, t, x_1, x_2, \Omega_2)$

FOR CONFIGURATIONS $y(r,t)$ ONLY THE DBI ACTION IS

$$\boxed{S = -T \int dr\, dt\; r^2 \left[1 + y'^2 - \frac{\dot{y}^2}{(r^2+y^2)^2} \right]^{1/2}}$$

STATIC SOLUTION NEAR THE AdS BOUNDARY

$$\boxed{y(r) \sim y_0 + \frac{\sigma}{r} + O\left(\frac{1}{r^2}\right)}$$

y_0 IS IN FACT THE MINIMUM LENGTH OF A (3,5) STRING

$\Rightarrow$ THE "BULK" FIELD $y(r,t)$ IS DUAL TO THE MASS OPERATOR IN THE 2+1 FIELD THEORY

$$\mathcal{O} \sim \bar{\psi}\psi + \cdots$$

$\Rightarrow$ $y_0 = m =$ MASS OF HYPERMULTIPLET FIELD.

$$\sigma = \langle \mathcal{O} \rangle$$

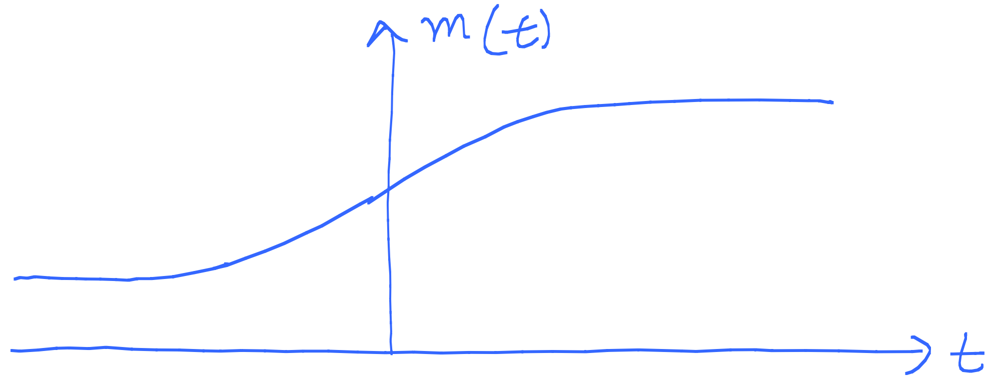
SUPPOSE WE WANT TO STUDY THE DUAL FIELD THEORY WITH A TIME DEPENDENT MASS TERM

$$m(t)$$

THIS MEANS THAT WE NEED TO FIND A SOLUTION OF DBI ACTION $y(r,t)$ WITH THE BOUNDARY CONDITION

$$y(\infty, t) = m(t)$$

IF $m(t)$ IS OF THE FORM



WE NEED TO IMPOSE AN APPROPRIATE INITIAL CONDITION TO ENSURE THAT THE STATE IS VACUUM AT EARLY TIMES

THE PROBLEM OF QUANTUM QUENCH IN THE FIELD THEORY REDUCES TO SOLUTION OF THE DBI EQUATIONS OF MOTION

WE WILL DESCRIBE A NUMERICAL SOLUTION LATER

.....

BUT LET'S FIRST DISCUSS AN ANALYTIC SOLUTION FOR A LOWER DIMENSIONAL BRANE.

OTHER BRANES

PROBE	WRAPS	DUAL CFT
D1	AdS_2	$0+1$
D3	$AdS_3 \times S^1$	$1+1$
D5	$AdS_4 \times S^2$	$2+1$
D7	$AdS_5 \times S^3$	$3+1$

PROBE BRANES HAVE BEEN USEFUL FOR DESCRIBING

- QUARKS IN $\mathcal{N}=4$ SYM (D7 BRANES)
- QUANTUM CRITICAL BEHAVIOR IN $2+1$, $1+1$ DIMENSIONS

ROTATING D1 BRANES

CONSIDER A SMALL NUMBER OF D1 (OR F1) BRANES IN $AdS_5 \times S^5$

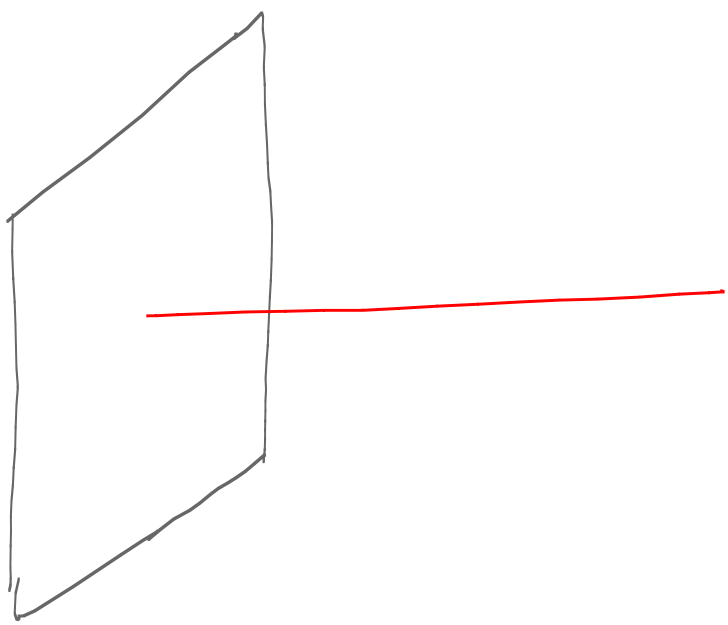
$AdS_5 \times S^5$ METRIC

$$\boxed{ds^2 = r^2(-dt^2 + d\vec{x}^2) + \frac{dr^2}{r^2} + d\theta^2 + \sin^2\theta\, d\varphi^2 + \cos^2\theta\, d\Omega_3^2}$$

THE 1-BRANE IS WRAPPED ALONG r DIRECTION

TRANSVERSE COORDINATES

$$\theta(r,t),\ \vec{x}(r,t),\ \varphi(r,t),\ \Omega_3(r,t)$$



WE WILL LOOK FOR CLASSICAL SOLUTIONS OF THE FORM

$$\vec{x} = 0, \quad \theta = \pi/2, \quad \Omega_3 = \text{pole}$$

$\varphi(t,r) \rightarrow$ ROTATING BRANE

THE DUAL OPERATOR TO $\varphi(t,r)$ IS AN INTERACTION TERM BETWEEN HYPERMULTIPLET FIELDS AND ADJOINTS

$$\int dt \sum_{i=1}^{2} \overline{Q_i} \left(\operatorname{Im} \Phi_3 \, e^{-i\varphi(t,\infty)} \right) Q_i$$

$\Phi_1, \Phi_2, \Phi_3 \rightarrow$ 3 COMPLEX SCALARS OF $N = 4$

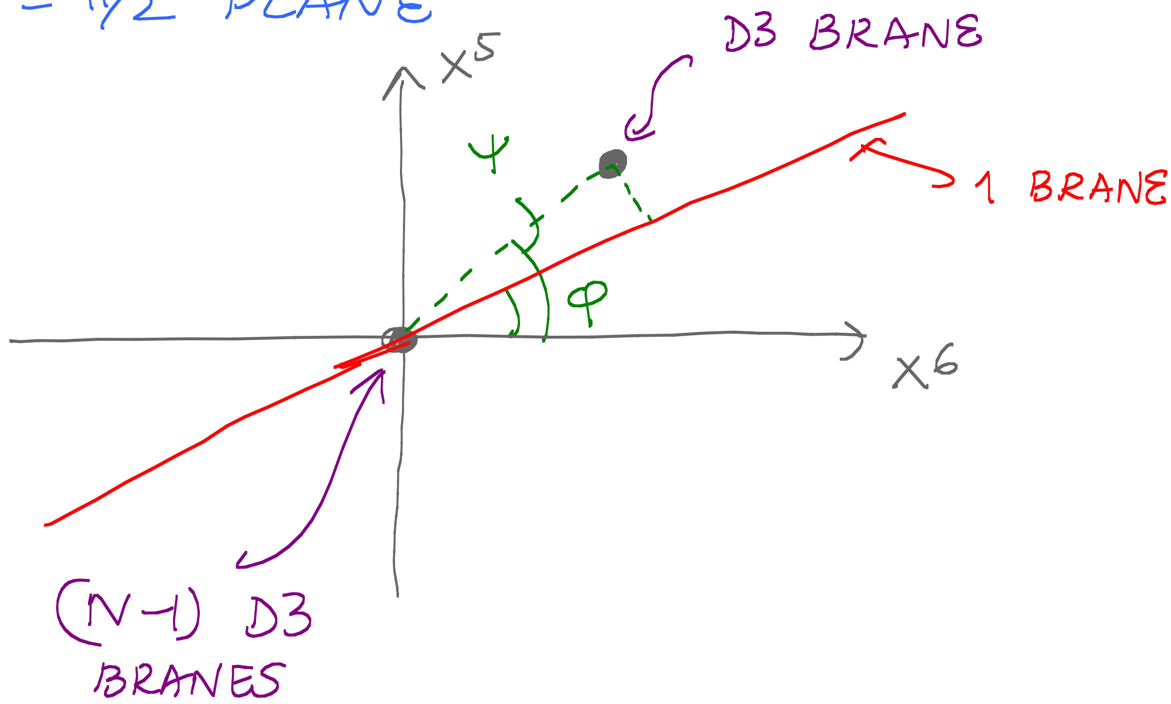
$Q_i \rightarrow$ 2 COMPLEX SCALARS IN FUNDAMENTAL

Φ_3 : PHASE ROTATION DESCRIBED BY φ

$$\theta = \pi/2 \qquad \Phi_1 = \Phi_2 = 0$$

CONSIDER A SOLUTION WHERE THE 1-BRANE PASSES THROUGH THE ORIGIN IN THE 6D TRANSVERSE PLANE

IN COLOUMB BRANCH, LET US SEPARATE ONE OF THE D3 BRANES TO A POINT ON THE $\theta = \pi/2$ PLANE



$$\text{MASS OF } Q_i = R \sin(\psi - \varphi)$$
$$= \mathrm{Im}(\Phi_3 e^{-i\varphi})$$

FOR SUCH CONFIGURATIONS CHOOSE EDDINGTON-FINKELSTEIN COOORDINATES IN $AdS_5 \times S^5$

$$\boxed{ds^2 = 2\,dr\,dv - r^2 dv^2 + d\theta^2 + \sin^2\theta\, d\varphi^2 + \cos^2\theta\, d\Omega_3^2}$$

THE ONLY WORLDSHEET FIELD IS

$$\varphi(r,v) = \varphi(u,v)$$

$$u \equiv v + \frac{2}{r}$$

EQUATION OF MOTION

$$\boxed{\partial_u \partial_v \varphi + \frac{2}{L} \partial_u \varphi\, \partial_v \left(\frac{\partial_u \varphi\, \partial_v \varphi}{r^2} \right) + \frac{2}{L} \partial_v \varphi\, \partial_u \left(\frac{\partial_u \varphi\, \partial_v \varphi}{r^2} \right) = 0}$$

THERE ARE SOLUTIONS

$$\boxed{\varphi(u)}$$

OR

$$\boxed{\varphi(v)}$$

THE SOLUTIONS $\varphi(v)$ ARE INGOING AT POINCARE HORIZON — WE WILL CHOOSE THEM.

THE INDUCED METRIC ON THE WORLDSHEET IS

$$\boxed{ds^2 = 2\,dr\,dv - \left[r^2 - (\partial_v \varphi)^2\right] dv^2}$$

THIS IS A VAIDYA METRIC

HAS AN APPARENT HORIZON AT

$$r = |\partial_v \varphi|$$

FLUCTUATIONS AROUND THIS SOLUTION WILL PERCEIVE THIS APPARENT HORIZON

y^I : FLUCTUATIONS IN THE TRANSVERSE DIRECTION LABELLED BY $I = 1 \dots 8$

$\gamma^{(0)}_{ab}$: INDUCED METRIC WITH THIS CLASSICAL SOLN.

$G_{IJ}(r, v, \varphi_0)$: TARGET SPACE METRIC

$$\boxed{S_2 = \frac{T}{2} \int dt\, dv \sqrt{-\gamma^{(0)}}\, \gamma^{(0)}_{ab}\, G_{IJ}\, \partial_a y^I \partial_b y^J}$$

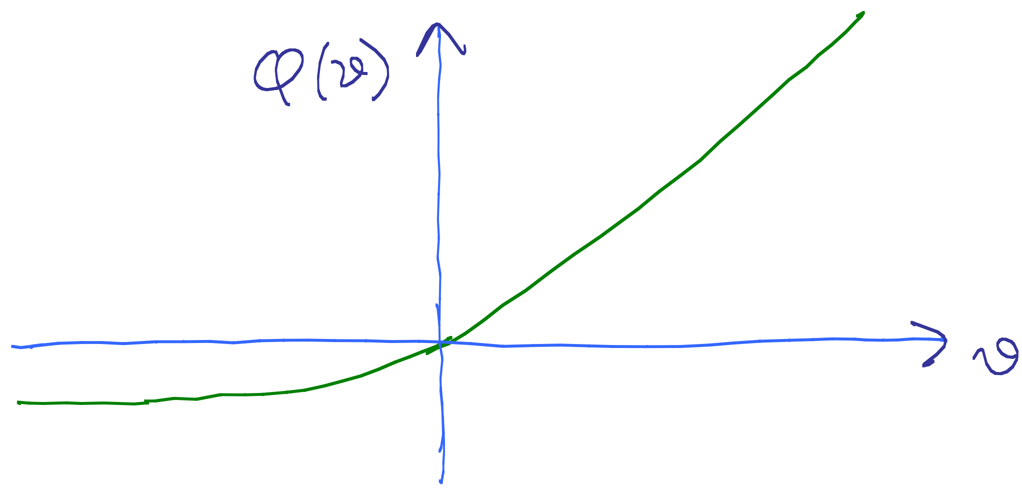
DUE TO THE LARGE REDSHIFT NEAR AN APPARENT HORIZON FLUCTUATIONS HAVE A THERMAL-LIKE SPECTRUM WITH A LOCAL HAWKING TEMPERATURE

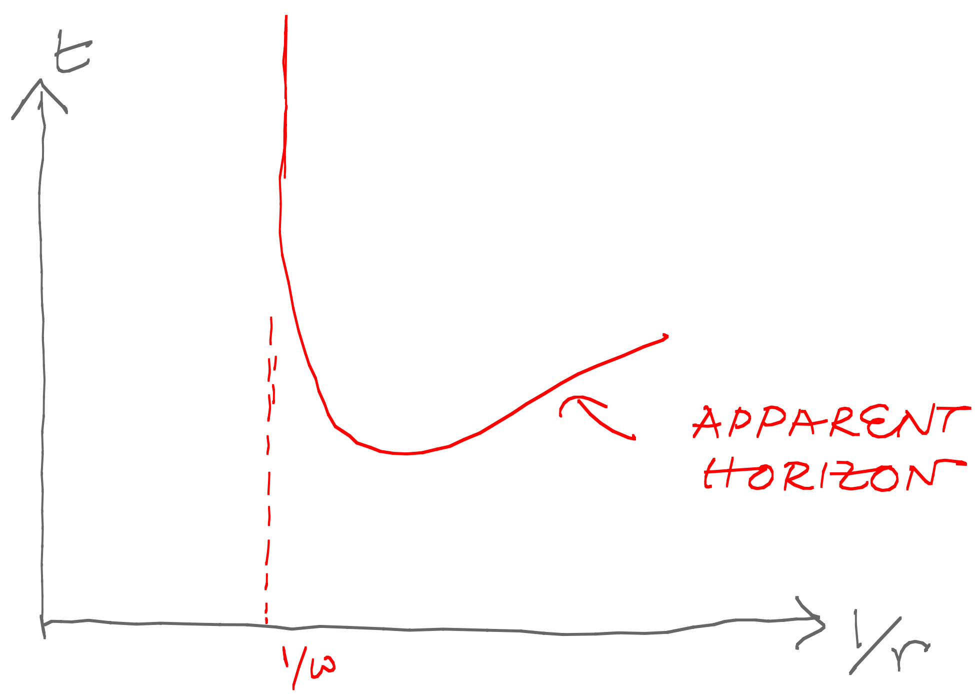
$$\boxed{T_H \sim \varphi'(v)}$$

- WHEN $\varphi(v)$ ASYMPTOTES TO

$$\varphi(v) \to \omega v \qquad v \to \infty$$

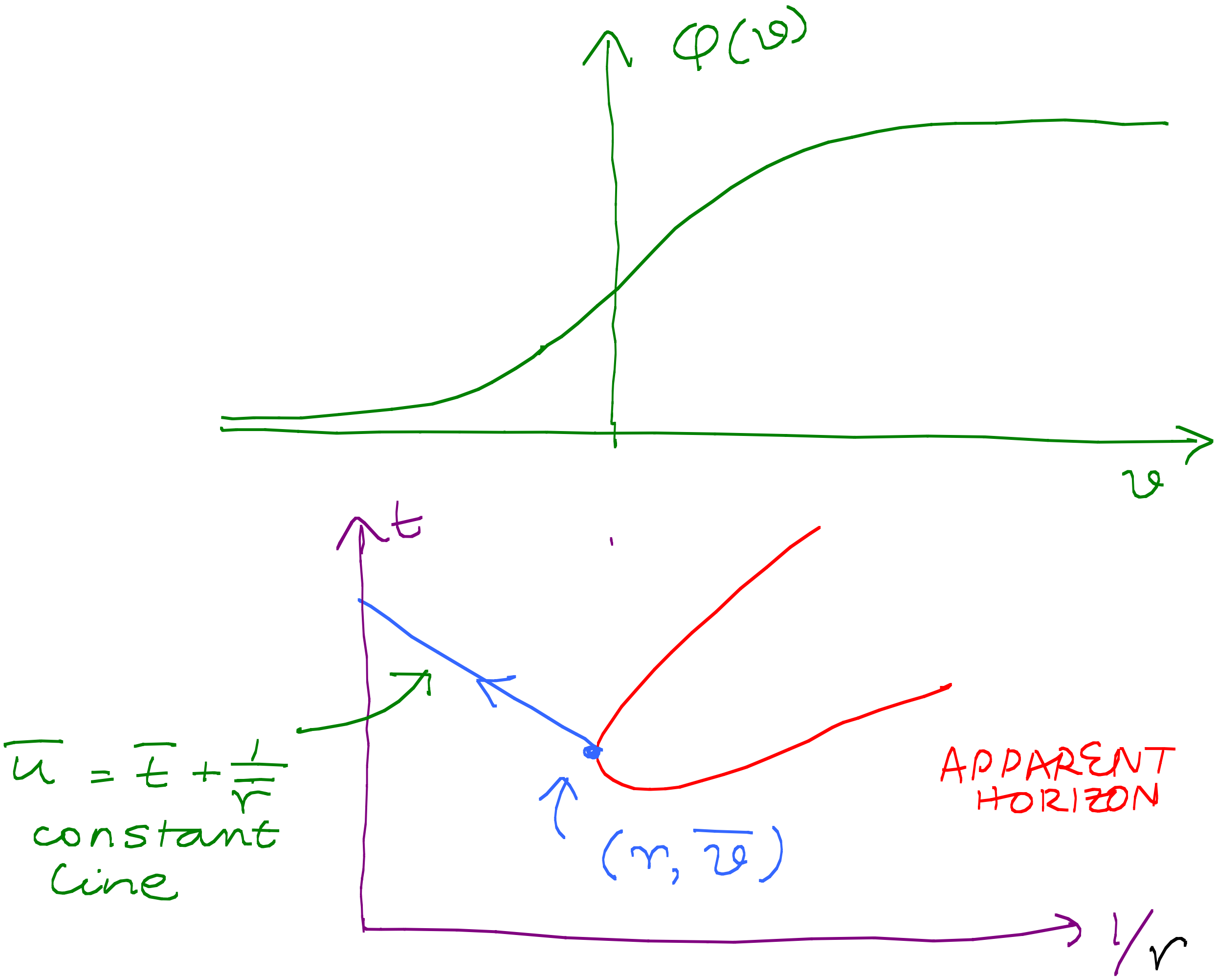
THE APPARENT HORIZON ASYMPTOTES TO AN EVENT HORIZON AT $r = \omega$





A THERMAL STATE AT LATE TIMES

HOWEVER WHEN APPROACHES A CONSTANT FOR LARGE v APPARENT HORIZON FORMS AND RECEEDS BACK



NOW SYSTEM REVERTS AT LATE TIMES — AT INTERMEDIATE TIMES APPROXIMATELY THERMAL

ON THE BOUNDARY WE NOW HAVE A "TIME-DEPENDENT TEMPERATURE"

IN AN EIKONAL APPROXIMATION THE TEMPERATURE ON THE BOUNDARY AT TIME t IS

$$T_H(t) = \varphi'(\bar{v})$$

WHERE $\bar{v}$ IS THE VALUE OF v AT THE INTERSECTION OF A BACKWARD LIGHT RAY STARTING FROM BOUNDARY AND THE APPARENT HORIZON

$$t = \bar{v} + \frac{2}{\varphi'(v)}$$

$$\to \frac{2}{\varphi'(v)} \quad \text{as } v \to \infty$$

$$\Rightarrow \qquad T_H(t) \sim \frac{2}{t} \qquad \text{FOR LATE TIMES}$$

THIS BEHAVIOR IS RELATED TO THE FACT THAT IN THE PROBE APPROXIMATION WE HAVE ESSENTIALLY AN OPEN SYSTEM

ENERGY IS INJECTED INTO THE BRANE — THIS CAN LEAK INTO THE BULK

ONLY THE HYPERMULTIPLET FIELDS BEHAVE THERMALLY

GLUONS OF $N=4$ — WHICH ARE DUAL TO SUPERGRAVITY MODES — REMAIN AT $T=0$ IN THE PROBE LIMIT

THUS TO HAVE A THERMAL SITUATION AT LATE TIMES ONE HAS TO CONSTANTLY INJECT ENERGY

THE SOLUTION

$$\varphi(\vartheta) = \omega \vartheta$$

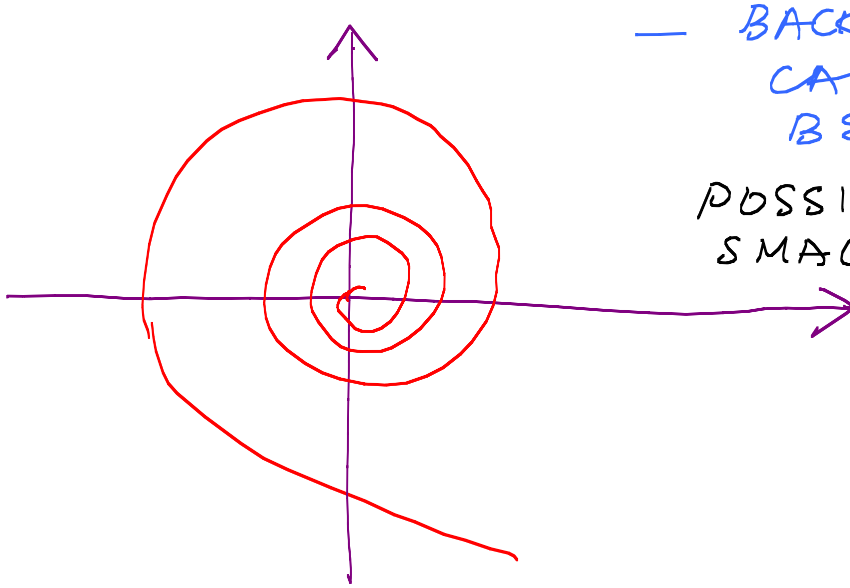
REPRESENTS A UNIFORMLY ROTATING 1-BRANE

THIS INVOLVES A CONSTANT FLOW OF ENERGY FROM THE BOUNDARY INTO THE POINCARE HORIZON

THE ENERGY DENSITY IS IN FACT DIVERGENT AT THE POINCARE HORIZON $r=0$

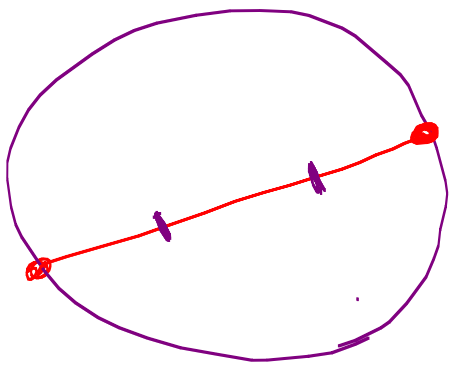
— BACKREACTION CANNOT BE IGNORED

POSSIBLY A SMALL BH



(SNAPSHOT OF ROTATING STRING)

HOWEVER, THERE ARE SOLUTIONS IN GLOBAL AdS WHICH HAVE BOUNDED ENERGY AND SMOOTH EVERYWHERE



GLOBAL AdS — FIXED TIME SLICE

HERE ENERGY FLOWS FROM ONE POINT OF BOUNDARY TO THE ANTIPODAL POINT

$$ds^2_{ind} = 2\,dr\,dv - (1+r^2-\omega^2)\,dv^2$$

HORIZON PRESENT ONLY IF

$$\omega > 1$$

→ A BLACK HOLE HORIZON FOLLOWED BY A WHITE HOLE HORIZON

THERMAL NATURE OF FLUCTUATIONS MANIFEST AS BROWNIAN MOTION OF END POINTS OF THE STRING

$$\boxed{\begin{aligned} &\langle 0|:\left[y^{\varphi}(t)-y^{\varphi}(t')\right]^2:|0\rangle \\ &\sim \frac{\pi(t-t')^2}{12\beta^2} \qquad \pi(t-t') \ll \beta \\ &\sim \frac{(t-t')}{2\beta}-\frac{1}{2\pi}\log\left[\frac{2\pi(t-t')}{\beta}\right] \\ &\qquad\qquad \pi(t-t') \gg \beta \end{aligned}}$$

SUCH BROWNIAN MOTION HAS BEEN OBSERVED FOR STRINGS IN AdS BLACK BRANE

— HERE THE BULK HORIZON INDUCES A HORIZON ON WORLDSHEET — HAWKING RADIATION ALONG BRANE (Martinec & Lawrence)

[(de Boer, Hubeny, Rangamani & Shigemori ;

Son & Teaney)

IN OUR CASE, THE BULK IS PURE AdS IN PROBE APPROXIMATION

THERMALITY ENTIRELY DUE TO MOTION OF THE STRING

IN THE BOUNDARY THEORY, A TIME DEPENDENT COUPLING LEADS TO THIS KIND OF THERMAL BEHAVIOR

SIMILAR PHENOMENA USED TO UNDERSTAND FAST THERMALIZATION IN MESON SECTOR OF $N=4$ (D7 BRANES) FOLLOWING RAPID CHANGE OF BARYON CHEMICAL POTENTIAL

(Hashimoto, Iizuka, Oka)

RESULTS FOR OTHER PROBE BRANES

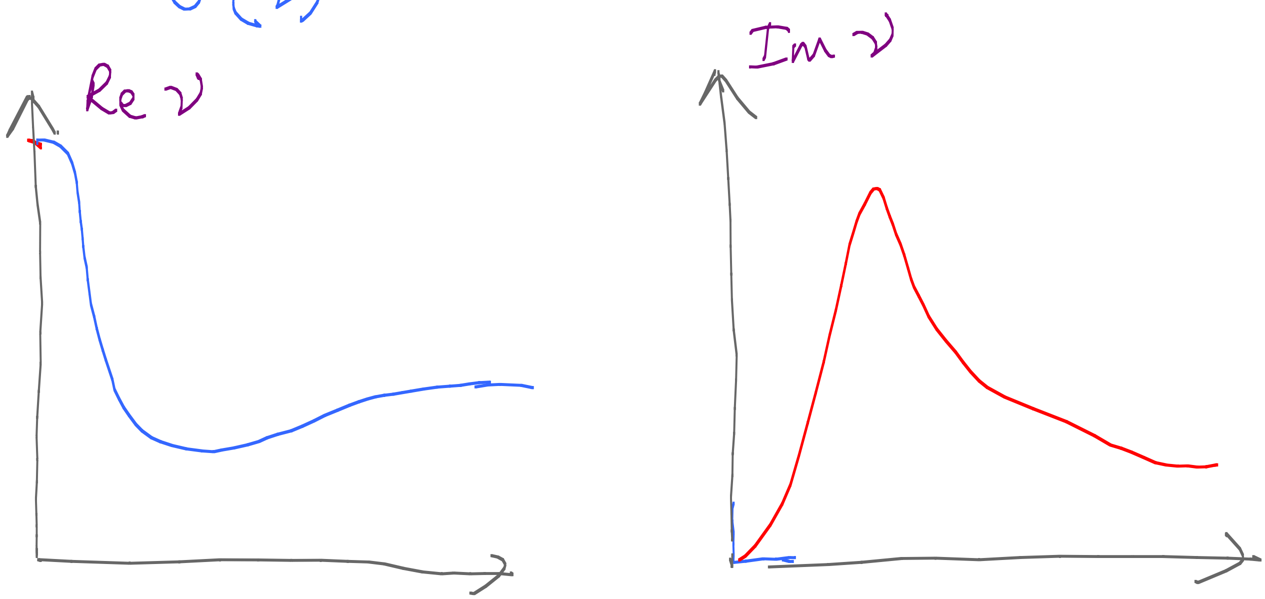
- D5 PROBES — FORMATION OF APPARENT HORIZON ON WORLDVOLUME DUE TO A QUANTUM QUENCH OF THE HYPERMULTIPLET MASS $m(t)$ — SEEN CLEARLY IN NUMERICAL CALCULATIONS

- D7 BRANES — HORIZON FORMATION DUE TO CONSTANT ROTATIONS — NUMERICAL

(UNIFORMLY ROTATING D5 SOLUTIONS OBTAINED EARLIER BY O'Bannon AND BY Evans & Threfall — HOWEVER EMERGENCE OF HORIZONS WERE NOT RECOGNIZED)

- D3 BRANES OBTAINED BY T-DUALITY FROM D1
  - NOW ONE CAN TURN ON A WORLDVOLUME ELECTRIC FIELD IN ADDITION TO ROTATION

CONSIDERING LINEARIZED FLUCTUATIONS OF A_x AROUND THIS SOLUTION ONE CAN CALCULATE THE CONDUCTIVITY $\sigma(\nu)$



DRUDE-LIKE BEHAVIOR

SUMMARY SO FAR

- IN EARLIER BULK CALCULATIONS THERMALIZATION OF A QFT DUE TO A TIME DEPENDENT COUPLING IS SIGNALLED BY BLACK HOLE FORMATION

- IN DEFECT CFT'S OBTAINED BY INTRODUCING D-BRANES AND IN PROBE APPROXIMATION THE BACKGROUND SUGRA SOLUTION IS UNCHANGED

  - NOW THERMALITY IS SIGNALLED BY FORMATION OF APPARENT HORIZONS ON THE WORLDVOLUME

THERE ARE SEVERAL OTHER PHYSICAL ASPECTS OF QUENCH WHICH CAN BE STUDIED IN THIS FRAMEWORK

— e.g. ENTANGLEMENT ENTROPY

— Talk by Takayanagi

QUENCH ACROSS ISOLATED CRITICAL POINT

In progress w P. Basu, A. Ghosh, K. Sengupta

QUANTUM QUENCH IS INTERESTING WHEN THE COUPLING PASSES THROUGH AN ISOLATED CRITICAL POINT

THEN THE RELAXATION OF THE INITIAL STATE CARRIES UNIVERSAL SIGNATURES OF THE CRITICAL POINT

PROBE BRANES ARE INTERESTING FROM THIS POINT OF VIEW – SINCE THERE ARE KNOWN EXAMPLES OF SUCH CRITICAL POINTS

CHIRAL SYMMETRY BREAKING TRANSITION IN D5-D3 SYSTEM

(Jensen, Karch & Son)

$AdS_5 \times S^5$ METRIC

$$\boxed{ds^2 = (r^2+y^2)(-dt^2+dx_1^2+dx_2^2+dx_3^2) + \frac{1}{r^2+y^2}\left(dr^2+r^2d\Omega_2^2+dy^2+y^2d\bar{\Omega}_2^2\right)}$$

D5 ALONG r, x_1, x_2, Ω_2

AS BEFORE WE WILL CONSIDER CLASSICAL SOLUTIONS WHICH EXCITE ONLY $y(r)$

NOW, IN ADDITION WE WILL TURN ON A WORLDVOLUME GAUGE FIELD

$$\boxed{A = A_t(r)\,dt + B x_2\, dx_1}$$

IN THE 2+1 CFT

$B \rightarrow$ NONDYNAMICAL MAGNETIC FIELD

$A_t(\infty) = \mu$ CHEMICAL POTENTIAL

$\Pi_t = \dfrac{\partial \mathcal{L}}{\partial A_t'} = \rho$ CHARGE DENSITY

Π_t IS A CONSTANT $\Rightarrow$ THE LEGENDRE TRANSFORMED ACTION

$$\boxed{S = -N\int dr\,\sqrt{1+y'^2}\,\sqrt{\rho^2+r^4+\frac{r^4B^2}{(r^2+y^2)^2}}}$$

NEAR $r=\infty$ $\psi = y/r$ IS A SCALAR IN AdS_4 WITH $m^2=-2$

THIS IS ABOVE THE BF BOUND

HOWEVER, NEAR $r=\infty$ THIS IS A SCALAR WITH

$$\boxed{m^2 = -\frac{2B^2}{(B^2+\rho^2)}}$$

IN AdS_2

THIS FALLS BELOW THE BF BOUND IN AdS_2 WHEN

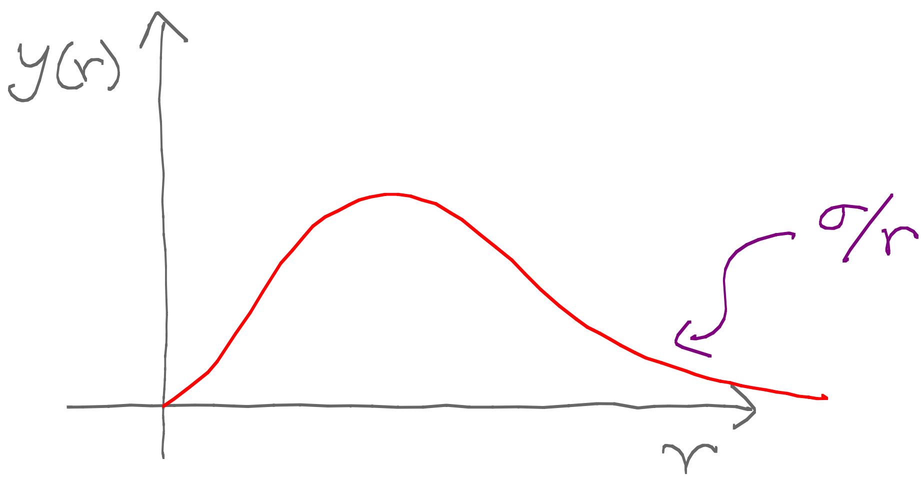
$$\boxed{B^2 > \frac{1}{7}\rho^2}$$

$\Rightarrow$ FOR $B^2 > 7\rho^2$ THE SOLUTION $y=0$ IS UNSTABLE

Jensen, Karch & Son OBTAINED A r DEPENDENT SOLUTION WITH

$$y(0) = 0$$
$$y'(\infty) = 0$$

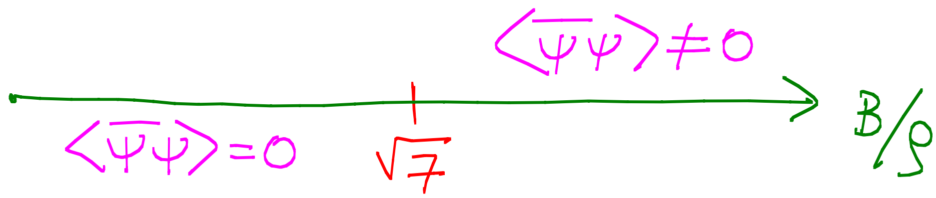
SINCE $y(\infty) = m$ IS THE MASS OF THE HYPERMULTIPLET FIELD THIS CORRESPONDS TO A MASSLESS THEORY



THE $1/r$ FALLOFF AT LARGE r MEANS THAT

$$\sigma = \langle \bar{\psi}\psi \rangle \neq 0$$

$\Rightarrow$ CHIRAL SYMMETRY IS SPONTANEOUSLY BROKEN



NEAR THE TRANSITION

$$\boxed{\langle\bar\psi\psi\rangle \sim \exp\left[-\frac{1}{\sqrt{7-\frac{\rho^2}{B^2}}}\right]}$$

$\Rightarrow$ BEREZINSKI - KOSTERLITZ - THOULESS

WE WANT TO PROBE THE NON-EQUILIBRIUM PHYSICS NEAR THIS TRANSITION

ONE POSSIBILITY IS TO INJECT QUARKS INTO THE THEORY FROM OUTSIDE AT SOME RATE — HENCE MAKE THE CHEMICAL POTENTIAL TIME DEPENDENT

Hashimoto, Iizuka & Oka for D7 probes

WE CONSIDERED PROBING THIS TRANSITION BY INTRODUCING A QUARK MASS $m(t)$

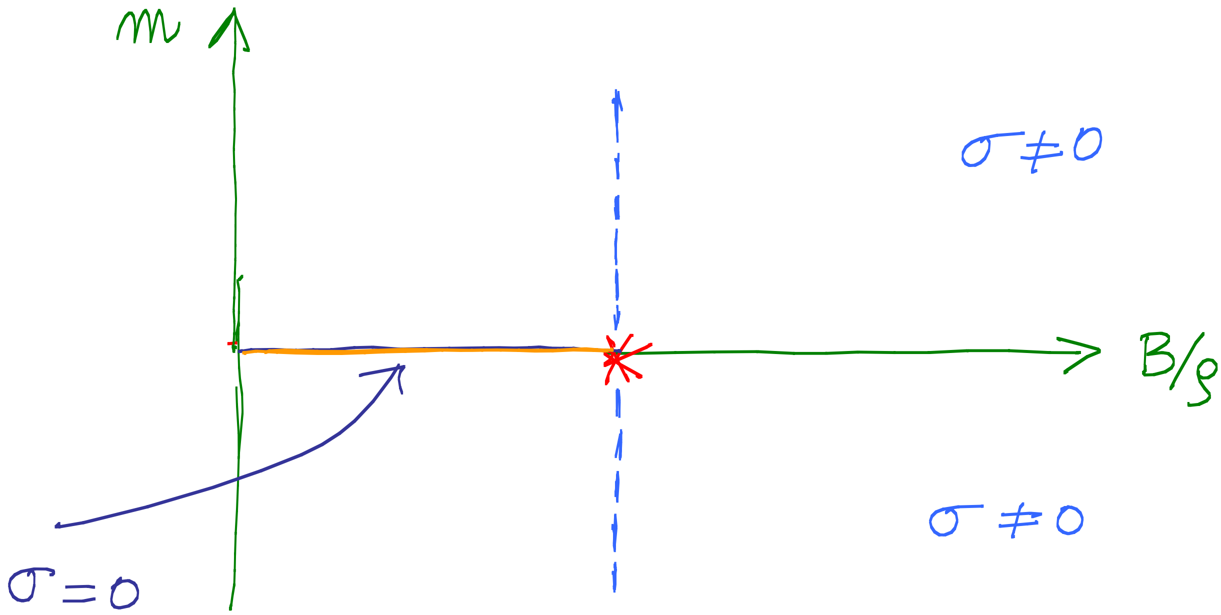
IN THE BRANE THEORY THIS MEANS WE CHANGE THE UV BOUNDARY CONDITIONS TO

$$\boxed{\lim_{r \to \infty} y(r,t) = m(t)}$$

FIRST CONSIDER THE STATICS OF THE PHASE TRANSITION

INTRODUCING A MASS EXPLICITLY BREAKS THE CHIRAL SYMMETRY AND $\langle \bar{\psi}\psi \rangle \neq 0$ ALWAYS

— THIS IS LIKE INTRODUCING AN EXTERNAL MAGNETIC FIELD IN A FERROMAGNET



AS WE TUNE m WE ENCOUNTER A PHASE TRANSITION IF WE CROSS

$$m = 0, \quad \frac{B}{\rho} = \frac{1}{\sqrt{7}}$$

WE HAVE CONSTRUCTED THE STATIC SOLUTION WITH THE MODIFIED BOUNDARY CONDITION ANALYTICALLY

$$\boxed{(m - \alpha\sigma) \sim - e^{-\left[\frac{\beta}{7 - \rho^2/B^2} + \gamma\,\frac{m+\alpha\sigma}{m-\alpha\sigma}\right]}}$$

$\alpha, \beta, \gamma \rightarrow$ NUMBERS

FOR THE CRITICAL VALUE OF THE MAGNETIC FIELD

$$\sigma \propto m$$

$$\Rightarrow \qquad \delta = 1$$

(AS OPPOSED TO $\delta = 3$ IN MFT)

THE TASK IS TO NOW FIND A TIME DEPENDENT SOLUTION WITH

$$\lim_{r \to \infty} y(r,t) = m(t)$$

FOR A SPECIFIED $m(t)$

OUR PREVIOUS DISCUSSION CARRIES OVER

⇒ INDUCED METRIC GENERALLY HAS APPARENT HORIZONS

FOR A SLOWLY VARYING $m(t)$ WE CAN SEE SIGNS OF FAILURE OF ADIABATICITY AS WE APPROACH THE CRITICAL POINT ALONG $B = \frac{1}{\sqrt{7}} \rho$

→ THIS IS SIGNALLED BY THE BREAKDOWN OF DERIVATIVE EXPANSION OF THE BULK SOLUTION FOR $y(r,t)$

IN CONCLUSION

- AdS/CFT HAS THE PROMISE OF PROVIDING A MUCH AWAITED NEW TOOL TO UNDERSTAND QUANTUM QUENCH

- THIS SHOULD PROVIDE NEW INSIGHT INTO THE APPROACH TO THERMALIZATION — WHICH IS AN IMPORTANT PROBLEM IN MANY AREAS OF PHYSICS

- WE ARE ALSO EXPLORING THIS IN PURELY BULK SETUPS — e.g. HOLOGRAPHIC SUPERCONDUCTING TRANSITIONS AT FINITE DENSITY

- THE HOPE IS THAT WE MAY LEARN SOME UNIVERSAL PHYSICS